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Full Length Research Paper

Anomalous modes in Faraday instability

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This research presents an experimental study on the Faraday instability in one-dimensional cells filled with a mixture of water and glycerol, and for a range forcing frequency between 10 and 60 Hz. It showed that for a particular forcing frequency, whose value depends on the width of the cell, an anomalous surface oscillation arises, that appears as a large wavelength mode oscillating subharmonically with respect to the normal modes predicted by the linear stability analysis. Since all others studied forcing frequency, the observed modes are in agreement with the ones theoretically predicted.

Key words: Faraday Instability, non-linear dynamics

PACS: 47.20.Dr, 47.20.Gv, 47.35.+i, 47.54.+r

INTRODUCTION

Faraday waves are excited when the free surface of a fluid layer is subjected to a periodic vertical acceleration (Faraday, 1831; Binks and van de Water, 1997; Kudrolli et al., 2001, Delon et al., 2009; Peña-Polo et al., 2014; Douady, 1990). When the acceleration exceeds a threshold value, surface waves appear oscillating at half the forcing frequency and with a critical wave number *k*. In the recent years, this system has been widely studied, both experimentally and theoretically, since it is a paradigm experiment for the investigation of pattern formation (Kumar and Tuckerman, 1994; Edwards and Fauve, 1993; Wernet et al., 2001), spatio-temporal phenomena (Epstein and Fineberg, 2004) and localized oscillations (Arbell and Fineberg, 2000).

Different patterns such as stripes, squares, and hexagons can be selected at instability onset depending on the properties of the liquid, such as its density ρ and kinematic viscosity ν, the depth of the liquid layer *h*, and the forcing frequency ω. It has been shown that in the case of relatively viscous liquids, with damp sidewall effects, the selected pattern is independent of the container shape. Since the boundary layer thickness at the free surface δ is of the magnitude order of $(\nu/\omega)^{1/2}$, then, the product *k* δ gives an estimation of the influence of viscous forces on the Faraday instability. When *k* δ <<1, viscous effects are weak, while *k*δ >>*1* implies that viscous effects are strong. Another important length scale in the problem is given by the liquid depth *h*. Indeed, the system behaviour is different depending if one is in the so-called deep-water limit (*kh > 1*) or the shallow water one (*kh < 1*). In the small viscosity (*k* δ <<1) and deep water (*kh > 1*) limits, in which we are interested in this paper, the deformation of the fluid surface can be described by normal modes obeying a Mathieu equation, that is, the system is analogous to coupled parametric oscillators (Benjamin and Ursell, 1954):

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$$\frac{d^2\zeta}{dt^2}+4\nu k^2\frac{d\zeta}{dt}+\left[gk+\frac{\sigma k^3}{\rho}-a\,k\cos(\omega t)\right]\zeta$$

Here, ζ represents the amplitude of the surface deformation, σ is the surface tension of the liquid against air, *a* the amplitude of the forcing acceleration and *g* the acceleration of gravity. The dependence of the critical wavelength on the forcing frequency is given by the dispersion relation:

$$\omega^2(k)=tanh(kh)\left[gk+\frac{\sigma k^3}{\rho}\right]$$

In the case of deep water limit, that is $kh > 1$, we have that $tanh(kh) \rightarrow 1$. When gk is dominant the mode is a gravity wave, and when $\sigma k^3/\rho$ dominates it is a capillary wave.

Here, we present an experimental study on the Faraday instability, where we are mainly interested to the low frequency forcing regime corresponding to gravity waves. The liquid is contained in strongly elongated rectangular cells, that is, cells with a large aspect ratio Γ between the length *l* and the width *d,* so that the excited waves may be considered as one-dimensional. We show that for some particular values of the forcing frequency and depending on the geometrical parameters of the cell, anomalous surface oscillations may appear. The wavelength of the anomalous mode is larger than the one predicted by the dispersion relation and the oscillation is subharmonic with respect to the ω/2 oscillation frequency predicted for normal modes. The nature of the anomalous mode is different from that of non-propagating solitons previously reported for the Faraday waves (Wu et al., 1984), even though chains of solitons of the same polarity could seem similar to the phenomenon here reported. The main difference is that in our case we do not observe localization of the oscillations. Indeed, once developed, the anomalous oscillation is an extended wave covering the entire fluid surface. Moreover, at difference with the behavior reported in Wu paper waves (Wu et al., 1984), we cannot create or delete individual solitons by applying a local perturbation. Another difference is that non propagating solitons exist also in annular resonator, while we do not observe the anomalous mode of oscillation in annular containers. Indeed, as we will see in the following, the origin of the anomalous mode of oscillation has to be searched in a strong resonance of the excited wave with the eigenmodes of the rectangular container, which takes place for a particular value of the forcing frequency at which the associated mode becomes two-dimensional. Similar resonances have been previously reported for a square cell (Jimenez, 1973).

EXPERIMENTAL SETUP

This experiment was performed with three different rectangular cells. Two cells have the same width, *d=1.4 cm*, but different lengths *l=13.7 cm* and *l=15.0 cm.* The third cell is narrower, *d=1 cm*, and has a length *l=15 cm*. The aspect ratio is Γ = 9.7, 10.7, and 15, for the cell 1, 2 and 3, respectively. For all the cells, the depth is *2 cm*. The bottom and the main structure of the cells are made of Aluminium. The lateral walls of the cells are made of transparent Plexiglass so that it is possible to visualize the fluid. Because of capillary forces, if the cell is not fully filled with the fluid, we observe a meniscus. The meniscus oscillation creates surface waves that can perturb the onset of the parametric instability (Douady and Fauve, 1998). In order to avoid such perturbation, the cell has always been fully filled. However, we checked that the presence of a meniscus does not change the nature of the observations described in the following. As working fluids we have used pure distilled water, different percentages of Glycerol in distilled water and ethanol. The room temperature is maintained at 20.0 ± 0.5°C, and we verified that the temperature of the fluid remains constant during the experiments. Once filled with the fluid, the cell is mounted over a mechanical vibration exciter *B&K* (model 4809). The vibration exciter is driven by a sinusoidal forcing delivered by a HP function generator and amplified by a 300 W power hi-fi amplifier and provides a clean vertical acceleration: the horizontal component is less than 1% of the vertical one.

A CCD camera is placed perpendicular to the main axis of the cell, in front of the transparent walls, in such a way to visualize the profile of the fluid free surface. The fluid is illuminated from above, and it is doped with black ink in order to avoid reflection from the bottom of the cell. Videos are recorded by means of a computer controlled frame grabber at 60 frames per second. By processing the recorded movies, we can measure the wavelength and the temporal oscillation frequency of the Faraday instability.

EXPERIMENTAL RESULTS AND INTERPRETATION

We measured the wavelength and the oscillation frequency of the Faraday instability in the range between *10* and *60 Hz*. The lower limit is due to our shaker acceleration range, the features of Faraday waves set the upper one. For most of the experiments, the temporal oscillation frequency of the surface waves is half the frequency of the sinusoidal forcing, as expected for parametric waves, and the wavelength corresponds to the one predicted by the linear stability analysis. However, for a very narrow range of frequency, that depends on the cell width and on the fluid properties, we have observed anomalous surface oscillations that appear as large adjacent bumps, oscillating in phase. As compared to Wu paper (Wu et al., 1984), the anomalous wave could appear as a chain of solitons of the same polarity. However, we have never observed either a single isolated soliton or a chain of solitons of opposite polarity, thus confirming our conjecture of an extended wave oscillation even though of different origin with respect to normal oscillations.

Figure 1a shows the wavelength of the surface oscillations measured as a function of the forcing frequency *f*, in the case of water and for the cell number

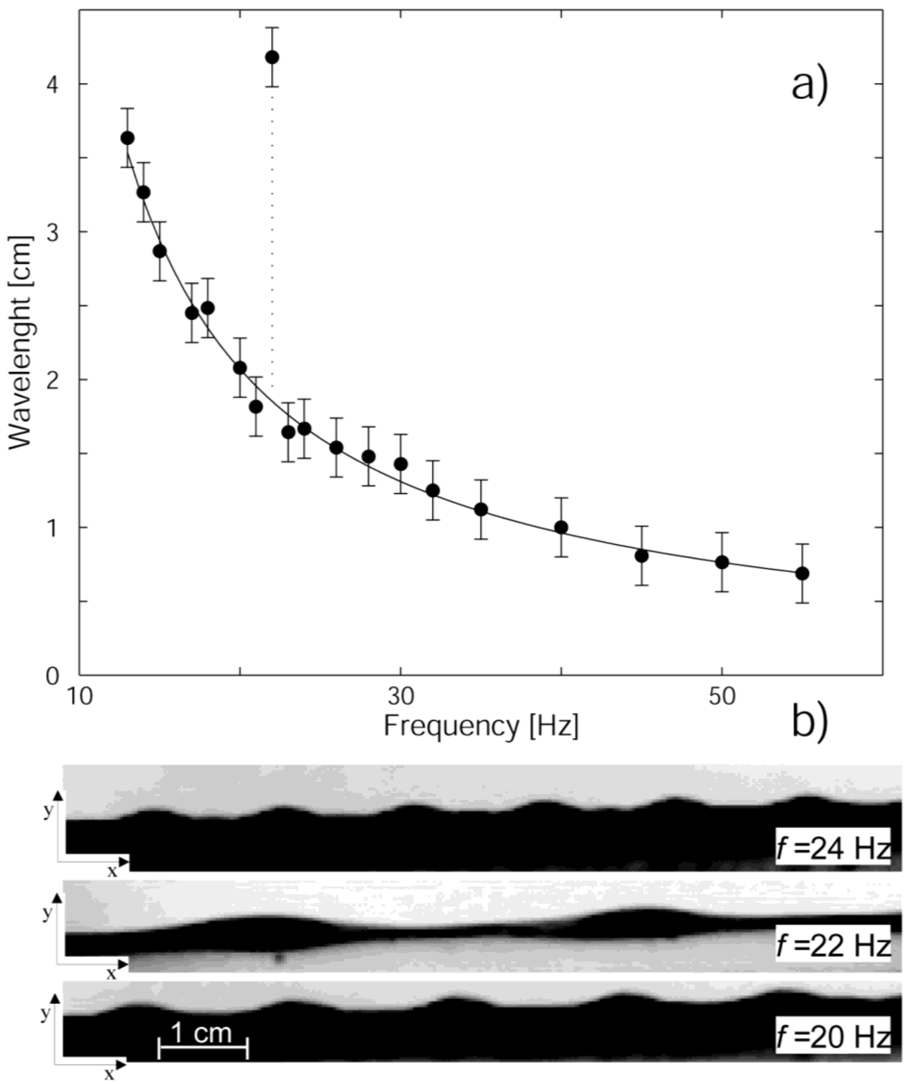

Figure 1. a) The wavelength of the surface wave (in cm) as a function of the driving frequency (in Hz) in the case of pure water and for the *15x1.4 cm* cell. The dots represent observed wavelengths and the line the dispersion relation predicted theoretically by the linear stability analysis. The bars correspond to the experimental incertitude on the measurement. b) Profile of the surface waves, projected on the XZ plane, for *f= 24, 22* and *20 Hz*.

1, *d=1.4 cm*, *l=13.7 cm*. The bars correspond to the experimental incertitude on the measurement. The dots represent the measured wavelengths λ and the line is the dispersion relation predicted theoretically by the linear stability analysis if we consider a one-dimensional mode, that is

$$z = z_0 \cos(k_x x)\cos(k_y y)\cos(2\pi f t)$$

with z_0 the unperturbed surface height, $k_X\ l=n\pi$ with *n=0,1,2,3,…* and $k_y\ d=q\pi$ with *q=1.* This is equivalent to assume that, while several wavelengths are present in the longitudinal direction, only half a wavelength fits along the width of the cell, and this for a large range of forcing frequencies. Under this approximation, the excited wavenumber $k=\sqrt{k_x^{\,2}+k_y^{\,2}}$ is almost equivalent to k_X, that is, the quantity that we have measured. It may be seen on Figure 1a that this approximation holds quite well for the whole range of forcing frequencies explored, except for a singular point, at f_a*=22 Hz*, that corresponds to the appearance of the anomalous oscillation. Correspondingly, the anomalous wavelength λ_a*=4.19±0.2 cm* is much larger that the one theoretically predicted (λ=*1.8 cm*).

In Figure *1b*, one can visualize the patterns observed

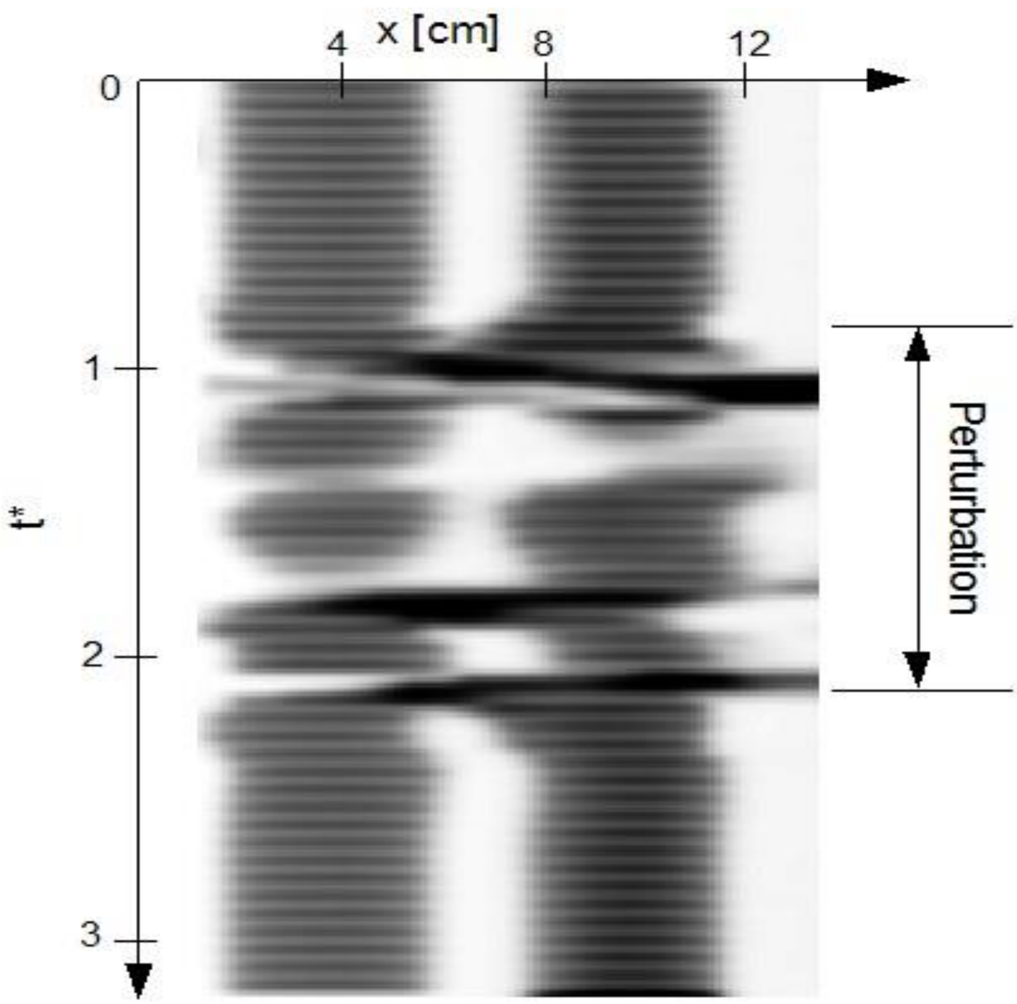

Figure 2. Spatio-temporal diagram of the wave amplitude obtained from the side view of the surface wave. t* is the time scaled by the driving frequency, x the length coordinate. The surface wave has been perturbed by blowing an air jet. After the end of the perturbation, the pattern re-became stable after only one oscillation of oscillator. Here, the fluid is pure water in the *13.7x1.4 cm* cell.

for forcing frequency below (*20 Hz*) and above (*24 Hz*) the anomalous frequency. If the same cell is filled with ethanol or with a *5%* water-Glycerol mixture, then an anomalous pattern is still observed at $f_a = 22$ *Hz*. However, the pattern wavelength depends on the fluid properties. Using a cell with the same width and a different length, cell number *2*, *d=1.4 cm* and *l=15 cm*, does not change the experimental findings. We have verified, by closing part of the cell with a transverse wall, that the properties of the anomalous wave do not depend on the cell length. However, the driving frequency f_a that generates the anomalous wave changes if one varies the width of the cell: the thinner the cell, the higher is the anomalous frequency. Indeed, for the cell number 3, *d=1 cm* and *l=15 cm*, we find f_a*=32 Hz*. If one uses a more viscous fluid, such as a 15% Glycerol in water, anomalous waves are not observed anymore for the whole range of forcing frequency explored (from 10 *up* to 60 *Hz*).

In order to study the stability of the anomalous waves, we perturbed the system in two different ways. In the first case, after the appearance of the anomalous wave, we blew a pressured air jet on the free surface of the fluid. As shown in the spatio-temporal diagram of Figure 2, the pattern is highly unstable while the air jet is blown, but, once the perturbation is stopped, it becomes stable again after a few cycle of the driving. Note that in the case of solitons such a perturbation would have led to the disappearance or to the creation of individual localized oscillations, which is not the case here. This is a further indication that the anomalous wave is a fully spatially correlated mode, involving the whole fluid surface.

The second stability test that we performed consists in

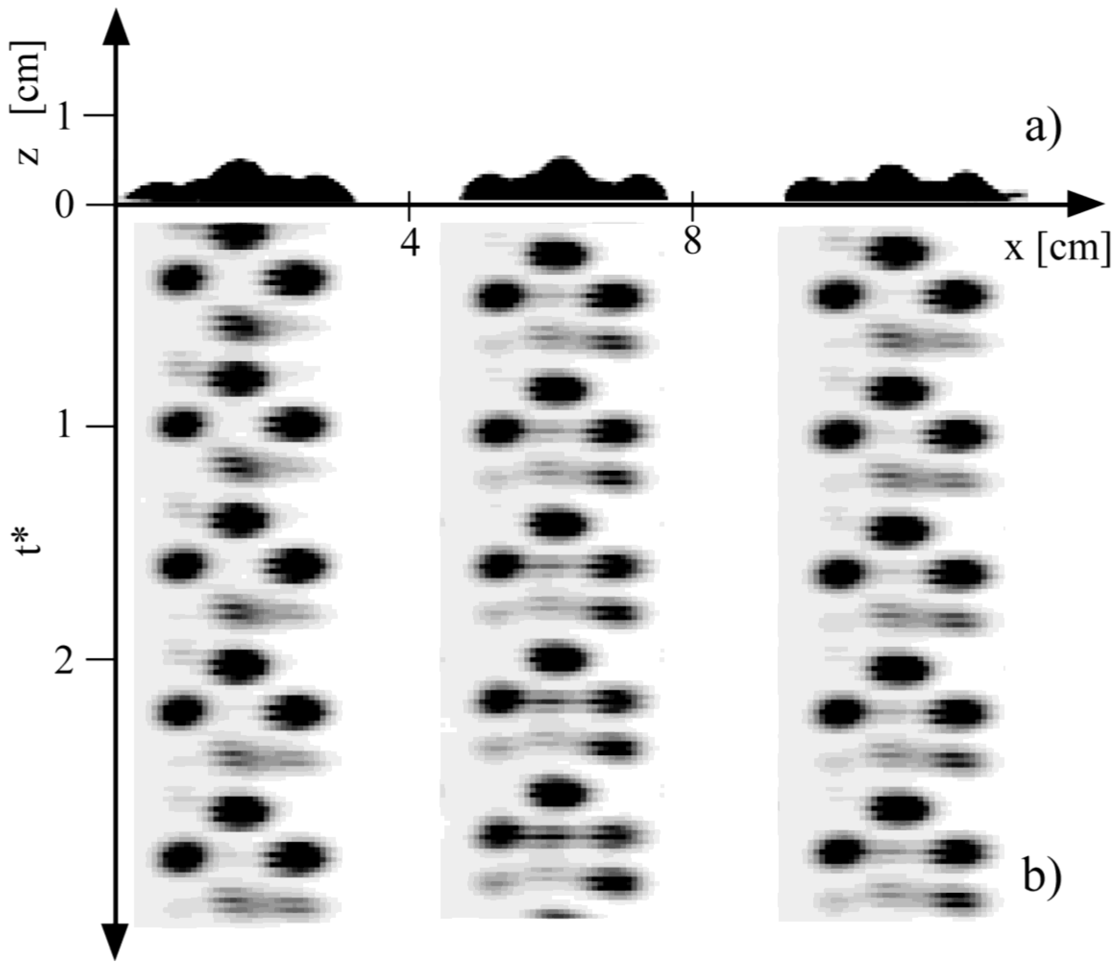

Figure 3. a) Profile of the surface waves, projected on the XZ plane, observed in the *15x1.4 cm* cell filled with pure water when simultaneously oscillated at f_a*=22 Hz* and f_S*=40 Hz*. x is the length coordinate, z the height of the wave. b) Spatio-temporal diagram of the surface wave profile;. $t^*=tf_a$ is the time scaled by the anomalous driving frequency.

forcing frequencies: f_a and f_2, where f_a represents the frequency that generates the anomalous waves of wavelength λ_a and f_2 the frequency that generates an instability pattern of wavelength λ_2. When the two frequencies are applied at the same time, the system shows the superposition of the two patterns with wavelength λ_a and λ_2, no matters which frequency is applied first and if the second mode is applied below or above the onset of the first one. Figure 3 shows (*a*) the profile and (*b*) the spatio-temporal diagram of the surface waves observed in the cell with width d=1.4 cm and filled with pure water when simultaneously oscillated at f_a *=22 Hz* and f_S*=40 Hz*. The incertitude on the measures of the onset of the instability does not allow to say if the onsets are varied by the presence of a second excitation mode.

As mentioned above, the anomalous mode it is not a meniscus effect. Indeed, it can be observed either if a meniscus is not present, that is, if the cell is perfectly filled, or when there is a negative or positive meniscus, respectively if the cell is not completely filled or if it is slightly too filled. However, the anomalous wave is very sensitive to frequency noise. In fact, the anomalous mode is observed only when the frequency bandwidth of the driving signal at f_a is less than a few *Hz*. If this is not the case, as happens for example by driving the cell with a low quality synthesizer, the anomalous wave does not appear. More than this, vibrating the cell with a large spectrum noise around the signal always leads to a pattern with the wavelength predicted by the linear stability analysis. The anomalous wave is not observed also when using an annular cell. In fact, we performed some analogous measurement on an annular *1 cm* width cell, having a diameter of *20 cm*. In this case, we didn't observe any surface wave with a wavelength significantly different from the one predicted by the linear stability analysis.

This observation also excludes the interpretation of the

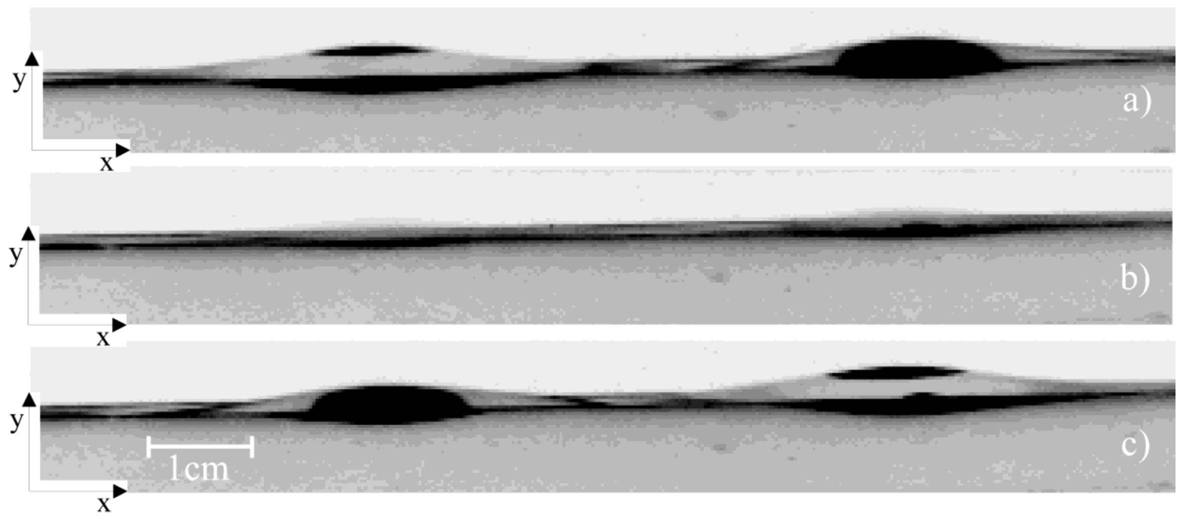

Figure 4. Side view (XZ plane) of the anomalous surface wave at three different times: a) t*=0, b) t*=1 and c) t*=2. The black spot on the bellies of the standing wave are the reflection of the illuminating light in the direction of the camera. The contrast is inversed after one period (t*=4). The fluid is pure water, the size of the cell is *14*x*1.4 cm*.

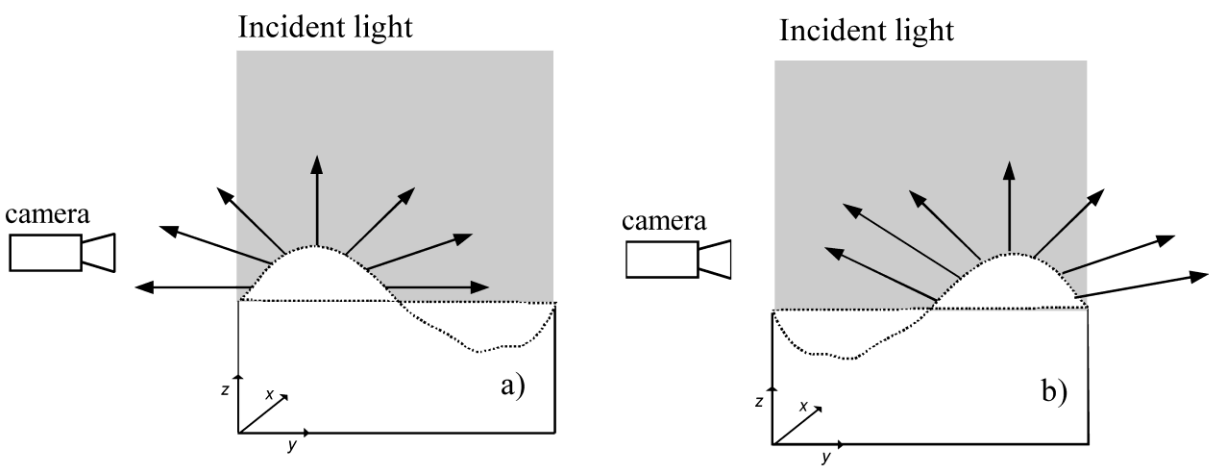

Figure 5. Transversal cut of the cell (ZY plane). Only the first part of the cell is illuminated (from above). a) When the surface wave has a positive slope in the left part of the cell, the light is reflected toward the camera. b) Only a little part of the light is reflected in the direction of the camera, when the surface presents a negative high in the left part of the cell.

anomalous waves in terms of chains of solitons, which are indeed reported for annular geometry.

The difference between the anomalous mode and the one predicted by the linear stability analysis is not only in the wavelength but also in the nature of the wave motion. In Figure 4 is shown a lateral view of the anomalous surface wave at three different times during a period of the driving signal. The black spot on the bellies of the standing wave are the reflection of the light that illuminates the cell from above. We can note that the reflected intensities from two adjacent bumps are not symmetrical. As schematically depicted in Figure 5, this can be interpreted, on a qualitative basis, by considering the different light that a bump scatters when it is close or far from the side of the cell where the CCD camera is placed. Indeed, when the surface wave has a positive slope close to this side of the cell, more light is reflected towards the camera (Figure 5a). On the contrary, when the slope of the wave is positive on the other side of the cell, less light is reflected in the direction of the camera (Figure 5b).

The dynamics of the black spots of Figure 4, alternating periodically during the time, suggest that the anomalous wave is a twisted and two- dimensional mode. Indeed, the oscillations occur in the direction of the width of the cell (perpendicularly at the wave axis) and the period of alternation between left and right belly is $T_{alt}=4/f_a$. For this reason, if one observes the wave from aside, he always sees a stationary wave with positive bellies.

DISCUSSION

The anomalous surface wave here reported cannot be associated to any of the pattern experimentally observed in previous experiments on Faraday instability. Our interpretation is that the anomalous wave has its origin in a strong resonance of the fluid with the lateral dimension of the cell. The main reasons that bring us to this conclusion are the following: first, the only geometrical parameter that affects the anomalous surface is the width of the cell. Indeed, changing the length of the cell or the depth of the fluid does not change the properties of the anomalous mode. Secondly, if the fluid is too viscous one cannot observe the anomalous mode. Since it has been showed that for viscous fluids the instability pattern is independent on the shape of the container (Edwards and Fauve, 1993; Wernet et al., 2001), one can think that if the fluid is too viscous, the dissipation damps the resonance.

By taking into account the experimental observations presented above, we can support our conjecture by considering the cell as a two-dimensional system and by decomposing the wavenumber of the excited mode, $k=\sqrt{k_x^{\,2}+k_y^{\,2}}$, in two components, a longitudinal one, $k_x l=n\pi$ with $n=0,1,2,3,\ldots$ and a transverse one, $k_y d=q\pi$, by taking $q=2$. This means that for some particular values of the forcing frequency, those giving rise to the anomalous oscillation, one full wavelength fits along the width of the cell. If this is the case, we can recalculate the wavenumber, for example in the case of the first cell filled with water. If we take the anomalous point on the wavelength curve of Figure 1a ($\lambda x = 4.19\pm0.2$ cm), this corresponds approximately to $k_x = 6\pi/15\ cm^{-1}$ whereas the transverse wavenumber is $k_y=\dfrac{2\pi}{1.4}\ cm^{-1}$. We thus obtain k=2.3 cm^{-1} which, in the limit of the experimental errors and of the rough estimate, can be considered fairly well consistent with the theoretical curve.

The two-dimensional character of the anomalous wave appears also in the twisted structure exhibited by the surface deformation, as shown in Figure 4. To explain the periodical alternation between left and right adjacent bumps, we can give the following qualitative argument. The transverse wavelength λy of the anomalous wave being approximately equal to the width d of the cell, we can assume λy=d and use the dispersion relation in the limit of deep water (kh>>1) to estimate the critical frequency $f_a=\dfrac{\omega_a}{2\pi}$ for the appearance of the anomalous mode:

$$f_a^{\,2}=\frac{g}{2\pi d}\left[1+\left(\frac{2\pi l_c}{d}\right)^2\right]$$

where $l_c=\sqrt{\dfrac{\sigma}{\rho g}}$ is the fluid capillary length, approximately *0.27 cm* for pure water. By substituting the values of d for the cell *1* and *2*, (d=1.4 and 1.0 cm), we find f_a = 11.8 and 15.2 *Hz*, respectively. These results are consistent with the experimental observations reported above.

Moreover, these frequencies fit quite nicely with the subharmonics of the driving oscillation frequency. This means that the anomalous mode can be considered as a two-dimensional twisted mode, oscillating sub-harmonically with respect to normal modes.

Conclusion

This research has shown that for some particular resonances with the lateral size of the cell, anomalous oscillations arise in rectangular geometry Faraday instability. We have shown that anomalous surface oscillations are a robust effect and cannot be assimilated to solitary waves. Anomalous oscillations are characterized by subharmonic spatial and temporal response with respect to normal modes. The study and the understanding of anomalous modes could be important in the design of pipeline or channels, in order to prevent anomalous waves from perturbing the functioning of the circuits (Jimenez 1973; Hsu and Kennedy, 1971). It would be then interesting to study these anomalous surface waves in other geometrical and physical conditions.

Conflict of interests

The author has not declared any conflict of interests.

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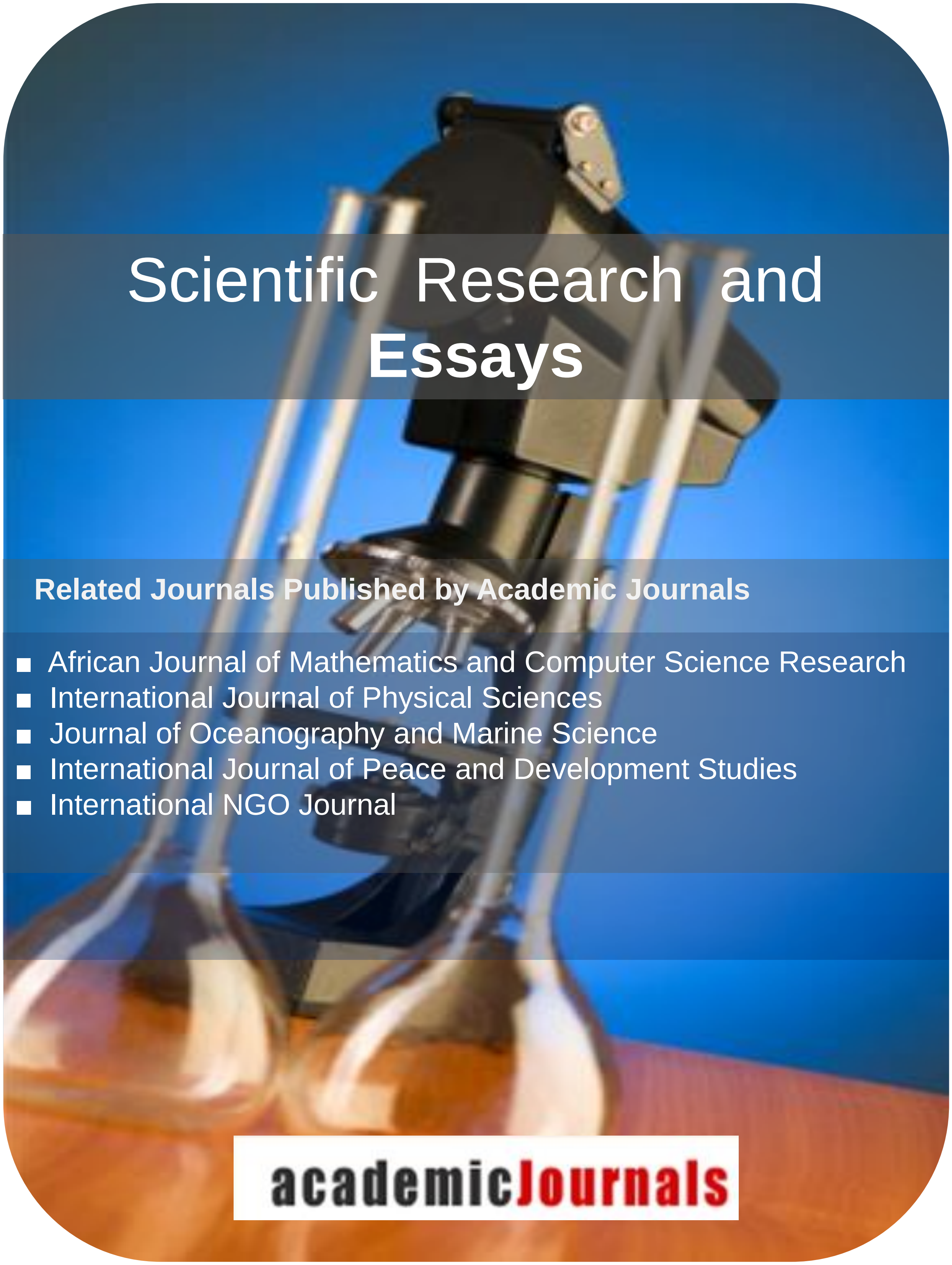
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